# Mistake in a Vector Problem


## Question

Given two points A(2, -1), B(0, 4) and the vector $\vec{u} = 3\vec{i} + k\vec{j}$ .If the

angle between $\vec{u}$ and $\vec{BA}$ is $45^o$, by considering $\vec{u} \cdot \vec{BA}$ find the values of k.

## Proposed solution

$\vec{BA} \cdot \vec{u} = (2\vec{i} - 5\vec{j}) \cdot (3\vec{i} + k\vec{j}) = 6\text{-}5k$

$\therefore 6 - 5k = \sqrt{2^2 + (-5)^2}\sqrt{9 + k^2}\cos 45^\circ$ …. (1)

$7k^2 - 40k - 63 = 0$ …. (2)

$(7k+9)(k-7) = 0$

$k = -\frac{9}{7}, 7.$ …. (3)

## Analysis

When $k = 7$, $\vec{u} = \cdot 3\vec{i} + 7\vec{j}$

$$\cos\angle\left(\overrightarrow{BA}, \vec{u}\right) = \frac{(2)(3) + (-5)(7)}{\sqrt{2^2 + (-5)^2}\sqrt{3^2 + 7^2}} = -\frac{1}{\sqrt{2}}$$

$\therefore \quad \angle\left(\overrightarrow{BA}, \vec{u}\right) = 135^\circ$ and not $45^\circ$.

$\therefore \quad k = 7$ should be rejected.

When $k = -\frac{9}{7}$, $\vec{u} = \cdot 3\vec{i} - \frac{9}{7}\vec{j}$

$$\cos\angle\left(\overrightarrow{BA}, \vec{u}\right) = \frac{(2)(3) + (-5)(-9/7)}{\sqrt{2^2 + (-5)^2}\sqrt{3^2 + (-9/7)^2}} = \frac{1}{\sqrt{2}}$$

$\therefore \quad \angle\left(\overrightarrow{BA}, \vec{u}\right) = 45^\circ$

$\therefore \quad k = -\frac{9}{7}$ is the only answer.

(Note : you can plot $\overrightarrow{BA}, \vec{u}$ for different values of $k$ to study the situation.)

## Lesson

Equation (2) is got by squaring equation (1).

**Squaring creates new redundant root !**